Homework Set 10

(sect 6.1 – 6.4)

Compute the quantities in questions 1 through 4 using the vectors below:

$$\boldsymbol{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \qquad \boldsymbol{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}, \qquad \boldsymbol{x} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$$

1. $\frac{1}{\boldsymbol{u} \cdot \boldsymbol{u}} \boldsymbol{u}$

2. $\left(\frac{\boldsymbol{x} \cdot \boldsymbol{w}}{\boldsymbol{x} \cdot \boldsymbol{x}} \right) \boldsymbol{x}$

3. $\|\boldsymbol{x}\|$

4. Find the distance between $\boldsymbol{w}$ and $\boldsymbol{x}$.

For questions 5 and 6, find a unit vector in the direction of the given vector.

5. $\begin{bmatrix} -30 \\ 40 \end{bmatrix}$

6. $\begin{bmatrix} 6 \\ -4 \\ -3 \end{bmatrix}$

For questions 7 through 10, determine which sets of vectors are orthogonal.

7. $\begin{bmatrix} 8 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

8. $\begin{bmatrix} 12 \\ 3 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}$

9. $\begin{bmatrix} -3 \\ 7 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -8 \\ 15 \\ -7 \end{bmatrix}$

10. $\begin{bmatrix} 2 \\ -7 \\ -1 \end{bmatrix}, \begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$

For questions 11 and 12, show that $\{\boldsymbol{u_1}, \boldsymbol{u_2}\}$ or $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$ is an orthogonal basis for $\mathbb{R}^2$ or $\mathbb{R}^3$, respectively. Then express $\boldsymbol{x}$ as a linear combination of the $\boldsymbol{u}$'s

11. $\boldsymbol{b_1} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \boldsymbol{b_2} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} -6 \\ 3 \end{bmatrix}$

12. $\boldsymbol{b_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \boldsymbol{b_2} = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}, \boldsymbol{b_3} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} 8 \\ -4 \\ -3 \end{bmatrix}$

13. Determine if the set of vectors is orthonormal. If the set is only orthogonal, normalize the vector to produce an orthonormal set. $\left\{\begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}, \begin{bmatrix} 1/3 \\ 2/3 \\ 0 \end{bmatrix}\right\}$

For questions 14 and 15, the given set is a basis for a subspace W. Use the Gram-Schmidt process to produce an orthogonal basis for W.

14. $\begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ -7 \end{bmatrix}$

15. $\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$